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Bremsstrahlung by high-energy particles in matter

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Abstract

We study bremsstrahlung by high-energy particles multiply scattered elastically in matter. It is shown that the formation of bremsstrahlung in the spectrum has a new mode, compared with the regime considered by Migdal (Migdal 1954 *Dokl. Akad. Nauk SSSR* **96** 49; Migdal 1956 *Phys. Rev.* **103** 1811) when the quantitative theory, the Landau–Pomeranchuk effect, in infinite matter was derived. In this mode, the emission spectrum depends sufficiently on the time the particle spends in the matter before photon emission. This results in the rearrangement of the spectrum in the short-wave region and leads to the nonlinear dependence of the spectral distribution of the bremsstrahlung on the time spent by the particle in the matter.

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1. Introduction

The influence of scattering in matter on bremsstrahlung by high-energy particles was first studied by Landau and Pomeranchuk [1, 2]. In these papers, they pointed to the suppression of the intensity of the bremsstrahlung due to the multiple elastic collisions of ultra-relativistic particles in matter; the Landau–Pomeranchuk effect. The quantitative theory of this effect has been derived by Migdal [3, 4]; the Landau–Pomeranchuk–Migdal (LPM) effect. The theory of the LPM effect has been developed further during research of the dispersion properties of scattering matter [5], its boundaries [6, 7], the Coulomb scattering of particles in matter [9–11] etc[1]. The key feature of the results obtained in the above-mentioned papers is the neglect of the influence of the multiple scattering of particles in matter on the photon production during the period preceding the photon emission. This is shown as the linear dependence of bremsstrahlung intensity on the time spent by the particle in the matter [1–11][2]. Such behaviour of the emission spectrum is reasonable when the matter is infinite rather than thick and confined in space, since in the latter case there is some interval of time

[1] A detailed review of the above-mentioned papers and other methods of calculating the LPM effect is given in [12].
[2] The exception is the extremely thin media [10, 11, 13–15] when the emission is generally a transition one in nature, which reduces progressively with the increase of the target thickness [6, 7, 10, 11].

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(the period from the entrance of the particle into the matter to the moment of the photon emission) which determines the state of the particle before the photon production. Therefore, at least quantitative estimations of the application of the methods developed in [3, 4] are necessary for the calculation of bremsstrahlung in the case of confined matter.

On the basis of the formalism of the two-particle Green's function in non-equilibrium matter [11], we obtain the probability of photon production by high-energy particles multiply scattered elastically in confined matter. If the scattering in the medium is rather weak, the spectrum coincides with that obtained in [3, 4] for infinite matter. When the scattering in matter is fairly strong, then another regime of bremsstrahlung formation takes place. In this case, the coherent time depends on the time spent by the particle in the matter before photon production. This leads to the rearrangement of the emission spectrum calculated in [3, 4]. It manifests itself in the dependence of the probability of the photon production on the parameters of the studied problem, which are the photon energy, the observation time, and the energy of the particle. In particular, the probability becomes the significantly nonlinear function of the observation time and the photon rate appears to be strongly suppressed in the short-wave region of the spectrum, compared with the spectral distribution obtained in [3, 4].

2. The coherence length and coherence time

Let us consider an ultra-relativistic particle ($E \gg m$, where E and m are its energy and mass) which enters the matter at the time $t = 0$ and undergoes multiple elastic collisions therein.

We start to study bremsstrahlung in matter by calculating the coherence time τ and coherence length l [6] for the photon emission. These are very convenient characteristics, which allow us to understand qualitatively the dynamics of bremsstrahlung formation in matter.

By definition, the coherence time is the period during which a photon leaves a particle for the distance of the order of the wavelength of the photon. This means ($h = c = 1$)

$$\tau - \frac{\vec{k}}{k}(\vec{r}(t+\tau) - \vec{r}(t)) \sim \frac{1}{\omega} \tag{1}$$

where ω and $\vec{k}$ are the energy and wave vector of the photon, and $\vec{r}(t)$ and $\vec{r}(t+\tau)$ are the radius vectors of the emitting particle at the moments when the photon production starts and finishes, respectively. When $h = c = 1$, the coherent length and the coherent time coincide, $l = \tau$.

Let us assume that the energy of the particle is large enough so that small-angle collisions take place in the matter and the influence of photon production on the particle scattering in the medium is negligible: $E \gg \omega$. We also assume that the particle only undergoes elastic collisions therein. In this case, its velocity is given by the standard formulae

$$\vec{v} = v_0\vec{e}_z\left(1 - \frac{\vec{\eta}^2}{2}\right) + v_0\vec{\eta} \qquad \vec{\eta} \perp \vec{e}_z \qquad |\vec{\eta}| \ll 1 \tag{2}$$

where v_0 is the particle velocity at the initial time $t = 0$ when it entered the matter along the OZ-axis.

If small-angle elastic scattering of the particle occurs in the matter, the value $\vec{\eta}(t)^2$ is given by the formula [1–5]

$$\vec{\eta}(t)^2 = \langle {\Theta_s}^2 \rangle t \tag{3}$$

where the parameter $\langle {\Theta_s}^2 \rangle$ is the mean square of the multiple scattering angle of the particle per unit path length [1–5].

To simplify the calculations we assume that the OZ-axis is directed along the vector $\vec{k}$. Then, substituting equation (2) into equation (1) we find

$$2\omega\gamma^{-2}\tau + \left\langle {\Theta_s}^2 \right\rangle k v_0 \tau^2 + 2\left\langle {\Theta_s}^2 \right\rangle k v_0 t\tau \sim 1 \tag{4}$$

where $\gamma = E/m = \left(1 - v_0^2\right)^{-1/2} \gg 1$. The formula determining τ shows that the coherent time depends in general on the moment t of the photon emission.

The solution of equation (4) leads to

$$\tau = \left(\frac{\omega}{\left\langle {\Theta_s}^2 \right\rangle k v_0 \gamma^2} + t \right) \cdot \left[\sqrt{1 + \frac{4\left\langle {\Theta_s}^2 \right\rangle k v_0}{\left(\omega\gamma^{-2} + \left\langle {\Theta_s}^2 \right\rangle k v_0 t\right)^2}} - 1 \right]. \tag{5}$$

It follows from this formula that there are two modes (or regimes) of photon emission by a high-energy particle. These modes differ from one another by the dependence (or independence) of τ on the time t spent by the particle in the matter before photon production, since the value of the parameter $\left\langle {\Theta_s}^2 \right\rangle t\gamma^2$ can be either large or small.

If the parameter $\left\langle {\Theta_s}^2 \right\rangle t\gamma^2 \ll 1$ is rather large we have ($k = \omega$)

$$\tau \sim \frac{1}{\sqrt{\left\langle {\Theta_s}^2 \right\rangle \omega}} \qquad \omega \ll \left\langle {\Theta_s}^2 \right\rangle \gamma^4 \tag{6}$$

$$\tau \sim \frac{\gamma^2}{\omega} \qquad \omega \gg \left\langle {\Theta_s}^2 \right\rangle \gamma^4. \tag{7}$$

The first case determined by equation (6) corresponds to the photon emission when the Landau–Pomeranchuk effect occurs [1, 2]. The fulfilment of condition (7) means that the radiation is formed due to the particle collisions with single scatterers in the matter. In this case, the emission spectrum is given by the Bethe–Heitler formula [16]. The key feature of the considered regime $\left(\left\langle {\Theta_s}^2 \right\rangle t\gamma^2 \ll 1\right)$, that of the bremsstrahlung formation, is that the coherent time does not depend on the time spent by the particle in the matter before the photon production.

In the second case, when $\left\langle {\Theta_s}^2 \right\rangle t\gamma^2 \gg 1$, we find

$$\tau \sim \frac{1}{\sqrt{\left\langle {\Theta_s}^2 \right\rangle \omega}} \qquad \omega \ll \left(\left\langle {\Theta_s}^2 \right\rangle t^2\right)^{-1} \tag{8}$$

$$\tau \sim \frac{1}{\omega\left\langle {\Theta_s}^2 \right\rangle t} \qquad \omega \gg \left(\left\langle {\Theta_s}^2 \right\rangle t^2\right)^{-1}. \tag{9}$$

It follows from formulae (8) and (9) that the regime of the bremsstrahlung formation at $\left\langle {\Theta_s}^2 \right\rangle t\gamma^2 \gg 1$ differs strongly from the mode studied in [1–12] and discussed above. Although the Landau–Pomeranchuk effect still takes place in such a mode in the long-wave region of the spectrum, the production of rather hard photons depends significantly on the time spent by the particle before the photon generation (see equation (9)).

The absence of such a regime of bremsstrahlung formation in infinite matter [1–4] is understandable since there is no moment of time which could be the original moment in the infinite medium. But, in the study of the emission in confined matter, such a moment takes place. This is the moment when the particle enters the matter.

The fact that the regime discussed above does not take place in the study of bremsstrahlung in confined media in [6, 7] can be explained in the following way. When the thickness of matter is rather large so that the particle undergoes multiple collisions therein, then the bremsstrahlung is formed both due to the crossing of the particle through the matter border (transition radiation) and owing to scattering of the particle inside the medium (bulk emission). Galitskii and Yakimetsl [6] and Gol'dman [7] have calculated the bulk emission using the method proposed by Migdal for infinite matter. The method [1–4] consists of the averaging of the general formula for the probability of photon production over all possible coordinates $\vec{r}(t_1)$; $\vec{r}(t_2)$ and velocities $\vec{v}(t_1)$; $\vec{v}(t_2)$ of the particle in the matter. Following Migdal, such averaging can be done by introducing the variables $\tau = t_1 - t_2$ (the coherent time) and $t = t_2$ and the conditional probability $\mathcal{P}(\vec{r}(t), \vec{v}(t), t; \vec{r}(t+\tau), \vec{v}(t+\tau), t+\tau)$. This is the probability that the coordinate and velocity of the particle are $\vec{r}(t+\tau)$ and $\vec{v}(t+\tau)$ at the moment $t+\tau$ provided that they are $\vec{r}(t)$ and $\vec{v}(t)$ at time t. As a result, in the case of rather soft photons ($\omega \ll E$) the probability of photon production $\mathrm{d}W$ can be written [3, 4] as follows

$$\mathrm{d}W = \frac{\alpha}{2\pi^2\omega}\mathrm{Re}\int_{-T}^{T}\mathrm{d}t\int_0^{T-t}\mathrm{d}\tau\,\mathrm{d}\vec{v}\,\mathrm{d}\vec{v}'\,\mathrm{d}\vec{r}\,\mathrm{d}\vec{r}'\cdot\mathcal{P}(\vec{r},\vec{v},t;\vec{r}',\vec{v}',t+\tau)(\vec{k}\times\vec{v})\cdot(\vec{k}\times\vec{v}')\cdot\exp(\mathrm{i}\omega\tau - \mathrm{i}(\vec{k}\vec{r}' - \vec{k}\vec{r}))\,\mathrm{d}^3\vec{k} \tag{10}$$

where α is the fine structure constant, and T is the time spent by the particle in the matter. Note that, although the variable t runs from $-T$ to T according to the direct changes of variables in integrals, it really goes from 0 to T since, by definition, the probability $\mathcal{P}(\vec{r}, \vec{v}, t; \vec{r}', \vec{v}', t+\tau)$ is equal to zero when t is negative.

The main approximation of Migdal's method is the factorization of the conditional probability $\mathcal{P}$

$$\mathcal{P}(\vec{r}(t), \vec{v}(t), t; \vec{r}(t+\tau), \vec{v}(t+\tau), t+\tau) = \mathcal{P}_1(\vec{r}(t), \vec{v}(t), t)\cdot\mathcal{P}_2(\vec{r}(t+\tau) - \vec{r}(t), \vec{v}(t+\tau) - \vec{v}(t), \tau) \tag{11}$$

where $\mathcal{P}_1$ is the probability that the coordinate and velocity of the particle are $\vec{r}(t)$ and $\vec{v}(t)$ at time t; $\mathcal{P}_2$ is the probability that they will differ from $\vec{r}(t)$ and $\vec{v}(t)$ by $(\vec{r}(t+\tau) - \vec{r}(t))$ and $\vec{v}(t+\tau) - \vec{v}(t))$ at a later time $t+\tau$.

The probability $\mathcal{P}_2(\tau)$ depends only on τ at any moment t but it does not depend on t itself, although t is the initial moment of time for the function $\mathcal{P}_2(\tau)$. The independence of $\mathcal{P}_2(\tau)$ from t is accounted for by the equivalence of all moments of particle motion in infinite matter, which does not occur in confined matter.

Note that the statement of the problem of bremsstrahlung in confined matter in itself means the setting of the initial condition on the border of the matter and not inside it. On the other hand, the solution of the considered problem demands the averaging of the required quantities over all possible coordinates and velocities of the particle in the matter. Indeed, this means that there is no fixed trajectory along which the particle moves. This circumstance does not allow us to transfer the origin inward from the border of the matter by means of moving along the particle trajectory or to reduce the problem of bremsstrahlung in infinite matter to that in a confined medium.

3. Two-particle Green's function and bremsstrahlung in matter

The probability of photon production by the current j^ν is given by the following expression [17]

$$\mathrm{d}^4w = 4\pi e_\mu e_\nu(1+n_\gamma)\delta\left(\omega^2 - \omega_k{}^2\right)\int\mathrm{d}^4x_1\,\mathrm{d}^4x_2\exp(-\mathrm{i}k(x_1 - x_2))\langle j^{\mu+}(x_1)j^\nu(x_2)\rangle\frac{\mathrm{d}^4k}{(2\pi)^3} \tag{12}$$

where $k = (\omega, \vec{k})$ and e_α are the four-vector and polarization vector of a photon, n_γ is the occupancy number of photons, and $j^\nu(x)$ is the particle current. The angled brackets represent the averaging over some states of particles in the matter; x are four-coordinates.

The bilinear combination of currents in the previous equation can be written as follows

$$\langle j^{\mu+}(x_1) j^{\nu}(x_2)\rangle = \langle i|(\hat{O}^{\mu})_{\alpha,\beta}((\hat{O}^{\dagger})^{\nu})_{\gamma,\delta}|j\rangle \left\langle \Psi^{\dagger}{}_{\delta}(x_1)\Psi_{\gamma}(x_1)\Psi_{\beta}(x_2)\Psi^{\dagger}{}_{\alpha}(x_2)\right\rangle \tag{13}$$

where $\langle i|(\hat{O}^{\mu})_{\alpha,\beta}((\hat{O}^{\dagger})^{\nu})_{\gamma,\delta}|j\rangle$ is the matrix element of some operator which is independent of the four-coordinates, $\Psi_{\alpha}(x)$ are the psi-operators in the Heisenberg picture, and α, β, γ and δ are the spin variables.

Thus, the problem of calculating bremsstrahlung in matter is reduced to that of obtaining the two-particle Green's function, which is proportional to the product of four Ψ-functions.

Let us assume that the influence of scattering in matter on the spin states of the particles is negligible. This is correct when the particles are ultra-relativistic particles at least [1–4]. Then, expanding the correlator $\left\langle \Psi^{\dagger}{}_{\delta}(x_1)\Psi_{\gamma}(x_1)\Psi_{\beta}(x_2)\Psi^{\dagger}{}_{\alpha}(x_2)\right\rangle$ over the whole set of plane waves, we can write the expression for the probability of photon production $\mathrm{d}W$ as follows (see equations (1)–(3))

$$\begin{aligned}
\mathrm{d}^4W = \overline{\mathrm{d}^4 w} = \frac{8\pi}{(2s+1)} \int \frac{\mathrm{d}^4k}{(2\pi)^3} \Bigg\{ &(1+n_\gamma)\delta\left(\omega^2 - \omega_k{}^2\right) \int_{-T}^{T} \mathrm{d}t_1 \int_{-T}^{T} \mathrm{d}t_2 \exp(\mathrm{i}\omega t_1 - \mathrm{i}\omega t_2) \\
&\times \int \mathrm{d}p_1^0\, \mathrm{d}p_2^0\, \mathrm{d}^4p_3\, \mathrm{d}^4p_4\, \delta^3(\vec{p}_3 - \vec{p}_1 - \vec{k}) \cdot \delta^3(\vec{p}_2 - \vec{p}_4 - \vec{k}) \\
&\cdot \exp\!\left(\mathrm{i}\left(p_3^0 - p_1^0\right) t_1\right) \exp\!\left(-\mathrm{i}\left(p_2^0 - p_4^0\right) t_2\right) \\
&\times \mathrm{Tr}\left[e_\mu e_\nu \langle i|(\hat{O}^{\mu})_{\alpha,\beta}((\hat{O}^{\dagger})^{\nu})_{\gamma,\delta}|j\rangle \overline{u}^{\alpha}(p_1)\overline{u}^{\beta}(p_2)u^{\gamma}(p_3)u^{\delta}(p_4)\right] \\
&\cdot K(p_4(+);\, p_2(-)|p_3(-);\, p_1(+)) \Bigg\}
\end{aligned} \tag{14}$$

where $p_i = \left(p_i^0, \vec{p}_i\right)$ are the four-momentum of the radiating particle, s is its spin, $u^{\alpha}(p)$ are spinors, and T is the observation time or the whole time spent by the particle in the matter. The line over d^4w represents the averaging and summing over the corresponding spin states of the particles in the matter. The function $K(p_4(+);\, p_2(-)|p_3(-);\, p_1(+))$ is the two-particle Green's function $K(4(+), 2(-), 3(-), 1(+))$ in the momentum representation

$$\begin{aligned}
K(p_4(+);\, p_2(-)|p_3(-);\, p_1(+)) = \int &\mathrm{d}X_1\, \mathrm{d}X_2\, \mathrm{d}X_3\, \mathrm{d}X_4 \exp(-\mathrm{i}p_1X_1 - \mathrm{i}p_2X_2 + \mathrm{i}p_3X_3 \\
&+ \mathrm{i}p_4X_4) \cdot \overline{u}(p_3)\overline{u}(p_4)u(p_1)u(p_2) \left\langle \Psi^{\dagger}{}_{\delta}(x_1)\Psi_{\gamma}(x_1)\Psi_{\beta}(x_2)\Psi^{\dagger}{}_{\alpha}(x_2)\right\rangle .
\end{aligned} \tag{15}$$

Thus, the problem of calculating bremsstrahlung in matter consists of obtaining the so-called non-chronological two-point Green's function $K(p_4(+);\, p_2(-)|p_3(-);\, p_1(+))$ in the momentum representation.

In the case of rather soft photons $\omega \ll E$, the matrix elements of the operator $\hat{O}$ can be easily found. After the calculation of the trace over all spin variables in equation (14) the square of $\hat{O}$ appears to be equal [3, 4, 17] to the product of the vector products of the wave vector and the particle momentum. As a result, we obtain from equation (14)

$$\begin{aligned}
\mathrm{d}W = \frac{\alpha}{4\omega\pi^2E^2} &\int_{-T}^{T} \mathrm{d}t_1 \int_{-T}^{T} \mathrm{d}t_2 (\vec{k}\times\vec{p}\,') \cdot (\vec{k}\times\vec{p}) \exp(-\mathrm{i}\omega t_1 + \mathrm{i}\omega t_2) \\
&\times F(\vec{p}\,' - \vec{k}/2;\, \vec{p}\,' + \vec{k}/2;\, t_2;\, \vec{p} + \vec{k}/2;\, \vec{p} - \vec{k}/2;\, t_1)\, \mathrm{d}^3\vec{k}
\end{aligned} \tag{16}$$

where α is the fine structure constant, and T is the observation time.

In obtaining the previous formula, the influence of the scattering on the mixing of the spin variables of the particle has been neglected, and we have also assumed that there is no photon bath in the matter ($n_\gamma = 0$). The function $F(\vec{p}\,' - \vec{k}/2;\, \vec{p}\,' + \vec{k}/2;\, t_2;\, \vec{p} + \vec{k}/2;\, \vec{p} - \vec{k}/2;\, t_1)$ is connected with $K(p_4(+); p_2(-)|p_3(-); p_1(-))$ introduced with equation (15) by using the expression

$$F(\vec{p}' - \vec{k}/2;\ \vec{p}' + \vec{k}/2;\ t_2\vec{p} + \vec{k}/2;\ \vec{p} - \vec{k}/2;\ t_1)$$

$$= \int \mathrm{d}p_1^0\,\mathrm{d}p_2^0\,\mathrm{d}^4p_3\,\mathrm{d}^4p_4\delta^3(\vec{p}_3 - \vec{p}_1 - \vec{k}) \cdot \delta^3(\vec{p}_2 - \vec{p}_4 - \vec{k}) \cdot \exp\left(\mathrm{i}\left(p_3^0 - p_1^0\right)t_1\right)$$

$$\times \exp\left(-\mathrm{i}\left(p_2^0 - p_4^0\right)t_2\right) \cdot K(p_4(+); p_2(-)|p_3(-); p_1(+))$$

$$\equiv F(\vec{p}';\ \vec{p}'t_2\vec{p};\ \vec{p};\ t_1). \quad (17)$$

4. Bremsstrahlung by ultra-relativistic particles in matter

Let us consider a fast ($E \gg \omega$; ω is the photon energy) ultra-relativistic particle ($m \ll E$; m is the particle mass, E is its energy) which enters the homogeneous amorphous matter at time $t = 0$, and undergoes multiple elastic collisions therein. We assume that the concentration of scatterers in the matter and the interaction of the particles with them are such that the following inequality takes place

$$n|f|^2 \cdot \max\{|f|; \lambda\} \ll 1 \quad (18)$$

where n is the number of particles per unit volume, f is the scattering amplitude of the particle by the centre in the matter, and λ is the wavelength of the particle.

This inequality corresponds to what is known as the gas approximation [18]. Then, to the first non-vanishing approximation with respect to the interaction of the particle and the concentration of the scattering centres of the matter, the function $F(\vec{p}';\ \vec{p}'t_2\vec{p};\ \vec{p};\ t_1)$ determined by equation (17) satisfies two kinetic-like equations[3]

$$\left(\frac{\partial}{\partial t_1} - \mathrm{i}\vec{k}\vec{v}\right) F_{\vec{k}}(\vec{p}';\ \vec{p}';\ t_2;\ \vec{p};\ \vec{p};\ t_1)$$

$$= \frac{(2\pi)^4 nm}{E(\vec{p})} \cdot \int \mathrm{d}^3\vec{q}|V(\vec{q})|^2 \cdot \{(F_{\vec{k}}(\vec{p}';\ \vec{p}';\ t_2;\ \vec{p}+\vec{q};\ \vec{p}+\vec{q};\ t_1)$$

$$- F_{\vec{k}}(\vec{p}';\ \vec{p}';\ t_2;\ \vec{p};\ \vec{p};\ t_1)) \cdot \delta(E(\vec{p}+\vec{q}) - E(\vec{p}))$$

$$+ [F_{\vec{k}}(\vec{p}';\ \vec{p}'+\vec{q};\ \vec{p}+\vec{q};\ t_2;\ \vec{p};\ t_1) \cdot \delta(E(\vec{p}') - E(\vec{p}'+\vec{q}))$$

$$- F_{\vec{k}}(\vec{p}';\ \vec{p}'-\vec{q};\ t_2;\ \vec{p};\ \vec{p}+\vec{q};\ t_1) \cdot \delta(E(\vec{p}')$$

$$- E(\vec{p}'-\vec{q}))] \cdot \cos(t_1(E(\vec{p}+\vec{q}) - E(\vec{p})))\} \quad (19)$$

$$\left(\frac{\partial}{\partial t_2} + \mathrm{i}\vec{k}\vec{v}'\right) F_{\vec{k}}(\vec{p}';\ \vec{p}';\ t_2\vec{p};\ \vec{p};\ t_1)$$

$$= \frac{(2\pi)^4 nm}{E(\vec{p})} \cdot \int \mathrm{d}^3\,\vec{q}|V(\vec{q})|^2 \cdot \{(F_{\vec{k}}(\vec{p}'+\vec{q};\ \vec{p}'+\vec{q};\ t_2;\ \vec{p};\ \vec{p};\ t_1)$$

$$- F_{\vec{k}}(\vec{p}';\ \vec{p}';\ t_2;\ \vec{p}+;\ \vec{p};\ t_1)) \cdot \delta(E(\vec{p}'+\vec{q})$$

$$- E(\vec{p}'))[F_{\vec{k}}(\vec{p}';\ \vec{p}'+\vec{q};\ t_2;\ \vec{p}+\vec{q};\ \vec{p};\ t_1) \cdot \delta(E(\vec{p}) - E(\vec{p}+\vec{q}))$$

$$- F_{\vec{k}}(\vec{p}';\ \vec{p}'+\vec{q};\ t_2;\ \vec{p};\ \vec{p}-\vec{q};\ t_1) \cdot \delta(E(\vec{p} - E(\vec{p}-\vec{q}))]$$

$$\cdot \cos(t_2(E(\vec{p}'+\vec{q}) - E(\vec{p}')))\}. \quad (20)$$

Here $E(\vec{p})$ is the energy of the particle having momentum $\vec{p}$ (the dispersion law is supposed to be known); $\vec{v} = \vec{p}/E(\vec{p})$; $\vec{v}' = \vec{p}'/E(\vec{p}')$ are the particle velocities; $V(\vec{q})$ is the Fourier

[3] A detailed derivation of these equations is presented in the appendix.

image of the potential of the interaction of the particle with the single scattering centre; n is the number of centres per unit volume.

It is clear that the solution of these equations depends essentially on the oscillating terms on the right-hand sides of equations (19) and (20) which contain the cosines.

First we consider the situation when the following inequality takes place

$$t_{1,2}|E(\vec{p}+\vec{q}) - E(\vec{p}\,'+\vec{q})| \ll 1. \tag{21}$$

To calculate the right-hand side of the inequality (21) we note that $\vec{p}$ and $\vec{p}\,'$ in this formula are the particle momentum before and after the photon emission, but $\vec{q}$ is the momentum transferred as a result of the individual collision of the particle in the matter.

Then we can write

$$t_{1,2}|E(\vec{p}+\vec{q}) - E(\vec{p}\,'+\vec{q})| \sim t\frac{(\vec{p}\vec{p}\,')\vec{q}}{p} \sim t\left(\left\langle{\Theta_s}^2\right\rangle\tau\right)^{1/2} q_\perp \ll t\tau\left\langle{\Theta_s}^2\right\rangle k \ll 1 \tag{22}$$

where $q_\perp$ is the momentum which is perpendicular to the direction of the particle movement, and $\left\langle{\Theta_s}^2\right\rangle$ is the mean square of the multiple scattering angle of the particle per unit path length, which has been introduced previously (see equations (2) and (3)).

It is seen from the last inequality that one is correct when the scattering in the matter is rather weak (see equation (5) and the text following it). In this way, the coherent time τ is given by equations (6) and (7) so that the mode of bremsstrahlung formation, which has been studied by Migdal [3, 4], takes place. On the other hand, the last inequality means that the cosines in the right-hand sides of equations (19) and (20) are approximately unit.

Let us expand the collision integrals in the formulae (19) and (20) over the small parameter $q/p \ll 1$. As a result, we derive the following equations for the function $F_{\vec{k}}(\vec{p}, \vec{p}\,', t_1, t_2)$:

$$\frac{\partial F_{\vec{k}}(\vec{p}, \vec{v}\,', t_1, t_2)}{\partial t_1} - \mathrm{i}\vec{k}\cdot\vec{v}\cdot F_{\vec{k}}(\vec{p}, \vec{p}\,', t_1, t_2) = \frac{\left\langle{\Theta_s}^2\right\rangle}{4}\cdot\left\{\frac{\partial^2}{\partial\vec{\eta}^2} + \frac{\partial^2}{\partial\vec{\eta}\partial\vec{\zeta}}\right\} F_{\vec{k}}(\vec{p}, \vec{p}\,', t_1, t_2) \tag{23}$$

$$\frac{\partial F_{\vec{k}}(\vec{p}, \vec{p}\,', t_1, t_2)}{\partial t_1} + \mathrm{i}\vec{k}\cdot\vec{v}\,'\cdot F_{\vec{k}}(\vec{p}, \vec{p}\,', t_1, t_2) = \frac{\left\langle{\Theta_s}^2\right\rangle}{4}\cdot\left\{\frac{\partial^2}{\partial\vec{\zeta}^2} + \frac{\partial^2}{\partial\vec{\eta}\partial\vec{\zeta}}\right\} F_{\vec{k}}(\vec{p}, \vec{p}\,', t_1, t_2). \tag{24}$$

In equations (23) and (24) the following notations are introduced:

$$\begin{gathered}
F(\vec{p}\,' - \vec{k}/2 + \vec{q};\ \vec{p}\,' + \vec{k}/2;\ t_2;\ \vec{p} + \vec{k}/2 - \vec{q};\ \vec{p} - \vec{k}/2;\ t_1) \equiv F_{\vec{k}}(\vec{p}, \vec{p}\,', t_1, t_2) \\
\vec{p} = p_0\vec{e}_z\left(1 - \frac{\vec{\eta}^2}{2}\right) + p_0\vec{\eta} \qquad \vec{\eta}\perp\vec{e}_z \qquad |\vec{\eta}| \ll 1 \\
\vec{p}\,' = p_0\vec{e}_z\left(1 - \frac{\vec{\zeta}^2}{2}\right) + p_0\vec{\zeta} \qquad \vec{\zeta}\perp\vec{e}_z \qquad |\vec{\zeta}| \ll 1 \\
\left\langle{\Theta_s}^2\right\rangle = 2mnp_0^{-2}E^{-2}\int \mathrm{d}^3\vec{q}q|V(\vec{q})|^2\delta(E(\vec{p}) - E(\vec{p}+\vec{q})) \\
\vec{v} = \frac{\vec{p}}{E} \qquad \vec{v}\,' = \frac{\vec{p}\,'}{E} \qquad |\vec{v}| = |\vec{v}\,'| = v_0 \qquad p_0 = |\vec{p}(t=0)|.
\end{gathered} \tag{25}$$

Furthermore, let us assume the variables $\tau \equiv t_1 - t_2$ (the coherent time); $t \equiv t_2$. After this, we sum equations (23) and (24). When the time of the photon production $\tau \ll t$, we can neglect τ in the equation which describes the evolution of the function $F_{\vec{k}}$ during the period $t \gg \tau$ which precedes the photon emission.

Then, we find

$$\frac{\partial F_{\vec{k}}(\vec{p}, \vec{p}\,', t, \tau)}{\partial \tau} - \mathrm{i}\vec{k} \cdot \vec{v} \cdot F_{\vec{k}}(\vec{p}, \vec{p}\,', t, \tau) = \frac{\langle {\Theta_s}^2 \rangle}{4} \cdot \left\{ \frac{\partial^2}{\partial(\vec{\eta} - \vec{\zeta})^2} \right\} F_{\vec{k}}(\vec{p}, \vec{p}\,', t, \tau) \tag{26}$$

$$\frac{\partial F_{\vec{k}}(\vec{p}, \vec{p}\,', t, \tau = 0)}{\partial t} - \mathrm{i}(\vec{k} \cdot \vec{v} - \vec{k} \cdot \vec{v}\,') \cdot F_{\vec{k}}(\vec{p}, \vec{p}\,', t, \tau = 0) = \frac{\langle {\Theta_s}^2 \rangle}{4} \cdot \left(\frac{\partial}{\partial \vec{\eta}} + \frac{\partial}{\partial \vec{\zeta}} \right)^2 F_{\vec{k}}(\vec{p}, \vec{p}\,', t, \tau = 0). \tag{27}$$

In obtaining these equations, the derivative with respect to the variable $\vec{\zeta}$ in equation (26) has been ignored because $F_{\vec{k}}$ is far more smooth a function of $\vec{\zeta}$ than the variable $(\vec{\eta} - \vec{\zeta})$.

Solving equations (26) and (27) under the initial condition

$$F_{\vec{k}}(\vec{p}, \vec{p}\,', t_1 = 0, t_2 = 0) = \frac{\delta(p - p_0)}{p^2} \cdot \frac{\delta(p' - p_0)}{p^2} \cdot \delta(\vec{\eta}) \cdot \delta(\vec{\zeta}) \tag{28}$$

we find the function $F_{\vec{k}}(\vec{p}, \vec{p}\,', t, \tau)$. Substituting $F_{\vec{k}}(\vec{p}, \vec{p}\,', t, \tau)$ into the formula (16) for the probability of photon production, we derive the result which coincides with the well-known formula obtained by Migdal [3, 4] for bremsstrahlung in infinite matter

$$\frac{\mathrm{d}W}{\mathrm{d}\omega} = \frac{\alpha T}{\pi \gamma^2} \int_0^{+\infty} \mathrm{d}z \exp\left(-\frac{z\left(\omega / \langle {\Theta_s}^2 \rangle\right)^{1/2}}{2\gamma^2} \right) \cdot \left\{ \sin\left(\frac{z\left(\omega / \langle {\Theta_s}^2 \rangle\right)^{1/2}}{2\gamma^2} \right) \coth z - \frac{\pi}{4} \right\} \tag{29}$$

where $\gamma = E/m$ is the Lorentz factor of an ultra-relativistic particle, and $k = \omega$.

The distinctive feature of the spectral distribution (29) is the linear dependence of the probability on the observation time T. This corresponds to the neglect of the influence of scattering matter on photon production during the time preceding the photon emission. Such a result is reasonable since the state of the particle is changed insignificantly due to scattering in the matter when the time preceding the photon production is rather small (see equations (21) and (22)).

In the second case, when the scattering in the matter is rather strong so that $t\tau\langle {\Theta_s}^2 \rangle k \gg 1$, the cosines in equations (19) and (20) are rapidly oscillating functions. Then, the terms containing these cosines in the right-hand sides of equations (19) and (20) are equal to zero. As a result, we have the following equations for the function $F_{\vec{k}}(\vec{p}, \vec{p}\,', t_1, t_2)$

$$\frac{\partial F_{\vec{k}}(\vec{p}, \vec{p}\,', t_1, t_2)}{\partial t_1} - \mathrm{i}\vec{k} \cdot \vec{v} \cdot F_{\vec{k}}(\vec{p}, \vec{p}\,', t_1, t_2) = \frac{\langle {\Theta_s}^2 \rangle}{4} \cdot \left\{ \frac{\partial^2}{\partial \vec{\eta}^2} \right\} F_{\vec{k}}(\vec{p}, \vec{p}\,', t_1, t_2) \tag{30}$$

$$\frac{\partial F_{\vec{k}}(\vec{p}, \vec{p}\,', t_1, t_2)}{\partial t_1} + \mathrm{i}\vec{k} \cdot \vec{v}\,' \cdot F_{\vec{k}}(\vec{p}, \vec{p}\,', t_1, t_2) = \frac{\langle {\Theta_s}^2 \rangle}{4} \cdot \left\{ \frac{\partial^2}{\partial \vec{\zeta}^2} \right\} F_{\vec{k}}(\vec{p}, \vec{p}\,', t_1, t_2). \tag{31}$$

Since the variables in equations (30) and (31) are independent but the initial condition (28) is the product of two functions of the same independent variables, the solution of the equations (30) and (31) with the initial condition (28) can be easily found. This is given by the formula

$$F_{\vec{k}}(\vec{p},\vec{p}',t_1,t_2)=\frac{\delta(p-p_0)}{p^2}\cdot\frac{\delta(p'-p_0)}{p'^2}\cdot\frac{a}{\pi\left\langle{\Theta_s}^2\right\rangle\sinh(at_1)}\cdot\exp\left\{-\frac{a}{\left\langle{\Theta_s}^2\right\rangle}\coth(at_1)\cdot(\vec{\eta})^2\right.$$
$$+\frac{2a}{\left\langle{\Theta_s}^2\right\rangle}\tanh\left(\frac{at_1}{2}\right)\cdot(\vec{\eta})\cdot(\vec{\theta})-\frac{2a}{\left\langle{\Theta_s}^2\right\rangle}\tanh\left(\frac{at_1}{2}\right)\cdot(\vec{\theta})^2+\mathrm{i}kvt_1\Bigg\}$$
$$\cdot\frac{b}{\pi\left\langle{\Theta_s}^2\right\rangle\sinh(bt_2)}\cdot\exp\Bigg\{-\frac{b}{\left\langle{\Theta_s}^2\right\rangle}\coth(bt_2)\cdot(\vec{\zeta})^2+\frac{2b}{\left\langle{\Theta_s}^2\right\rangle}\tanh\left(\frac{bt_2}{2}\right)$$
$$\left.\cdot(\vec{\zeta})\cdot(\vec{\theta})-\frac{2b}{\left\langle{\Theta_s}^2\right\rangle}\tanh\left(\frac{bt_2}{2}\right)\cdot(\vec{\theta})^2-\mathrm{i}kvt_2\right\}\tag{32}$$

where $a, b, v, \vec{\theta}$ are given by the expressions

$$a=\frac{(1+\mathrm{i})\sqrt{kv\left\langle{\Theta_s}^2\right\rangle}}{2}\qquad b=\frac{(1-\mathrm{i})\sqrt{kv\left\langle{\Theta_s}^2\right\rangle}}{2}\qquad v=\frac{p_0}{E(\vec{p}_0)}$$
$$\vec{k}=k\vec{e}_z\left(1-\frac{\vec{\theta}^2}{2}\right)+k\vec{\theta}\qquad\vec{\theta}\perp\vec{e}_z\qquad|\vec{\theta}|\ll1.\tag{33}$$

Both the statement of the problem and the initial condition (28) mean that the particle begins to move in the matter at time $t=0$. On the other hand, it moves as a free particle before this moment. To take this fact into account, we rewrite equation (16) as follows [7]

$$\mathrm{d}W=\frac{\alpha}{4\pi^2\omega E^2}\int_0^T\mathrm{d}t_1\int_0^T\mathrm{d}t_2\int\mathrm{d}^3\,\vec{p}\int\mathrm{d}^3\vec{p}'(\vec{k}\times\vec{p})\cdot(\vec{k}\times\vec{p}')\cdot\{\exp(\mathrm{i}(t_1-t_2))$$
$$\cdot[F_{\vec{k}}(\vec{p},\vec{p}',t_1,t_2)+F_{\vec{k}}(\vec{p},\vec{p}',t_1,t_2;a=b=0)]$$
$$-2\mathrm{Re}[F_{\vec{k}}(\vec{p},\vec{p}',t_1,t_2;a;b=0)\cdot\exp(\mathrm{i}(t_1-t_2))]\}\,\mathrm{d}^3\vec{k}\tag{34}$$

where α is the fine structure constant, and T is the observation time (the time spent by the particle in the matter).

Substituting $F_{\vec{k}}$ given by equation (32) into equation (34) and carrying out the necessary integrations, we derive

$$\frac{\mathrm{d}W}{\mathrm{d}\omega}=\frac{\alpha\omega}{2\pi\gamma^2}\mathrm{Re}\left\{\int_0^T\mathrm{d}t_1\int_0^T\mathrm{d}t_2\exp\left(-\frac{\mathrm{i}(t_1-t_2)\omega}{2\gamma^2}\right)\left[\frac{2}{\left(\frac{a}{\left\langle{\Theta_s}^2\right\rangle}\tanh at_1-\frac{\mathrm{i}\omega t_2}{2}\right)\cosh^2at_1}\right.\right.$$
$$\left.-\frac{1}{\left(\frac{a}{\left\langle{\Theta_s}^2\right\rangle}\tanh at_1+\frac{b}{\left\langle{\Theta_s}^2\right\rangle}\tanh bt_2\right)\cosh^2at_1}\right]$$
$$\left.+\frac{2\alpha a}{\pi\omega}\int_0^T\mathrm{d}t\exp\left(-\frac{\mathrm{i}t\omega}{2\gamma^2}\right)\left[\frac{1}{at}-\frac{1}{\sinh at\cosh at}\right]\right\}\tag{35}$$

where $k=\omega$; $\gamma=E/m$ is the Lorentz factor of an ultra-relativistic particle.

Let us analyse the probability of the photon production (35). In the range of the soft photons $\omega\ll\left\langle{\Theta_s}^2\right\rangle^{-1}T^{-2}$ the probability is given by the formula

$$\frac{\mathrm{d}W}{\mathrm{d}\omega}=\frac{\alpha T}{\pi}\sqrt{\frac{\left\langle{\Theta_s}^2\right\rangle}{\omega}}.\tag{36}$$

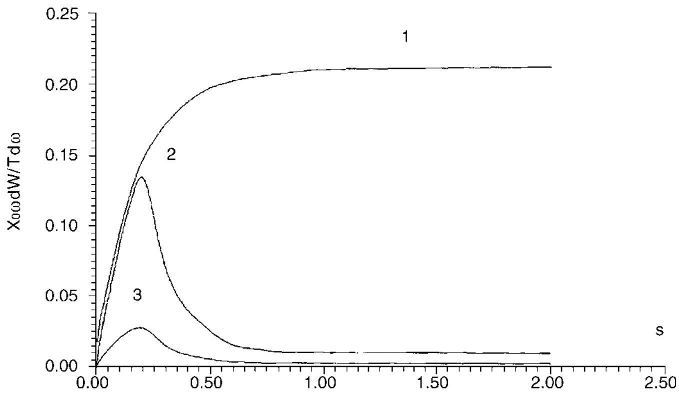


Figure 1. 1; the spectrum according to the Migdal's formula; 2, 3; the present results: 2; T = 0.1 X_0; 3; T = 0.5 X_0; $S^2 = \alpha\omega X_0$.

This expression coincides with the results obtained by Migdal in the long-wave range of the emission spectrum in infinite matter. This takes place because very soft photons are formed over the whole trajectory of the particle motion in the matter. Therefore, the influence of multiple collisions on photon production during the time preceding the photon generation is negligible. As a result, the probability is the linear function of the observation time T for such energies of the photons.

In the intermediate range of the spectrum $\langle {\Theta_s}^2\rangle^{-1} T^{-2} \ll \omega \ll \langle {\Theta_s}^2\rangle \gamma^4$ we have

$$\frac{\mathrm{d}W}{\mathrm{d}\omega} = -\frac{2\alpha}{\omega} \ln\left(\frac{1}{2\gamma^2}\sqrt{\frac{\omega}{\langle {\Theta_s}^2\rangle}}\right). \tag{37}$$

This formula coincides within a small term with the expression obtained by Pafomov [8] for transition radiation. In this way, we should note that the formula derived by Pafomov should be added to the Migdal result [3, 4] to obtain the total radiation, while equation (37) gives the total bremsstrahlung itself. This means that, in the intermediate range of the spectrum, the bremsstrahlung is formed due to the crossing of the particle through the matter border but the radiation arising from the movement of the particle inside the matter appears to be suppressed.

The probability of the production of the hard photons, so that $\omega \gg \langle {\Theta_s}^2\rangle \gamma^4$, is given by the expression

$$\frac{\mathrm{d}W}{\mathrm{d}\omega} = \frac{\alpha}{\pi\omega}\left(\frac{3}{8} + \frac{16\langle {\Theta_s}^2\rangle \gamma^8}{\omega}\right). \tag{38}$$

The dependence of the probability of photon production (35) is shown in figure 1.

It follows from equations (35)–(38) that the emission spectrum in such a mode (when scattering in the matter is so strong that $t\tau\langle {\Theta_s}^2\rangle k \gg 1$) is rearranged in comparison with Migdal's formula (see equation (29)). This manifests itself in the dependence of the probability of the photon production both on the observation time T and on the photon energy ω. Firstly, the probability dW is not a linear function of the observation time[4]. Besides, dW depends logarithmically on the photon energy in the range of rather hard photons. Finally,

[4] The nonlinear dependence of the probability of photon production obtained in [10, 11] is connected with taking into account the border of confined matter.

the probability of photon production is strongly suppressed in the short-wave region of the spectrum (see figure 1) compared with the mode studied by Migdal [3, 4]. The source of the suppression is the breaking of the coherence of photon production due to the collisions of the particles in the matter before the photon production. As a consequence of the scattering of the particle in the medium, its velocity appears inside some cone of the scattering angles. This means that the particle can be found with some probability inside this cone and can emit a photon when its velocity is one of a number of velocities inside the cone. If the apex angle of the cone is so large that $t\tau\left\langle{\Theta_s}^2\right\rangle k \gg 1$ (see also the text before equation (8)) this leads to the breaking of the coherence of the photon production since the velocities inside the cone are independent. This is the reason that, in the case of rather strong scattering, the collisions of the particles in the matter break the coherence of the photon emission more dramatically than in the situation of weak scattering matter $\left(t\tau\left\langle{\Theta_s}^2\right\rangle k \ll 1\right)$ when the emission spectrum is determined by Migdal's formula (29).

5. Conclusion

On the basis of the diagram formalism for the two-particle Green's function, we study the bremsstrahlung by ultra-relativistic particles multiply scattered elastically in confined matter. The probability of the bremsstrahlung is found. It is shown that, in the case of rather weak scattering matter, when $t\tau\left\langle{\Theta_s}^2\right\rangle k \ll 1$, the emission spectrum is given by the formulae of [3, 4]. This inequality determines the application limits of the results of Migdal's theory of bremsstrahlung by ultra-relativistic particles in the case of confined matter.

When the scattering in the matter is fairly strong so that $t\tau\left\langle{\Theta_s}^2\right\rangle k \gg 1$ the emission spectrum is rearranged (see equations (35)–(38) and figure 1) compared with that given by equation (29), and corresponds to the rather weak scattering of the particles in the medium $t\tau\left\langle{\Theta_s}^2\right\rangle k \ll 1$. This rearrangement manifests itself in the dependence of the probability of the photon production on all parameters of the considered problem: the observation time, the photon energy and the energy of the particle. The probability becomes the sufficiently nonlinear function of the time spent by the particle in the matter. It also depends logarithmically on the photon energy in the intermediate range of the emission spectrum. In addition, the probability of photon production appears to be strongly suppressed in the short-wave region of spectrum.

Appendix

We use the formalism of the two-particle Green's functions [19] for the derivation of the equation for the correlator $\langle \Psi^{\dagger}{}_{\delta}(x_1)\Psi_{\gamma}(x_1)\Psi_{\beta}(x_2)\Psi^{\dagger}{}_{\alpha}(x_2)\rangle$ determining the probability of photon production in scattering matter.

Following [19], we introduce the standard K, anti-chronological $\tilde{K}$, non-chronological K' and partial chronological $\hat{K}$ two-particle Green's functions in matter

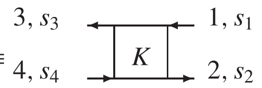
$$\begin{aligned}
K_{\delta,\beta;\gamma,\alpha}(x_4, x_2; x_3, x_1) &\equiv K(4,2;3,1) = \left\langle \hat{T}\left\{\hat{\Psi}_3\hat{\Psi}_4\hat{\Psi}_1^+\hat{\Psi}_2^+\right\}\right\rangle \\
\tilde{K}_{\delta,\beta;\gamma,\alpha}(x_4, x_2; x_3, x_1) &\equiv K(4,2;3,1) = \left\langle \hat{\tilde{T}}\left\{\hat{\Psi}_3\hat{\Psi}_4\hat{\Psi}_1^+\hat{\Psi}_2^+\right\}\right\rangle \\
K'_{\delta,\beta;\gamma,\alpha}(x_4, x_2; x_3, x_1) &\equiv K(4,2;3,1) = \left\langle \left\{\hat{\Psi}_3\hat{\Psi}_4\hat{\Psi}_1^+\hat{\Psi}_2^+\right\}\right\rangle
\end{aligned} \tag{39}$$

where the operators T and $\tilde{T}$ denote the chronological ordering of the field operators from left to right and from right to left, respectively. The angled brackets correspond to the averaging over some arbitrary state of the particles in the matter. This state can also be nonstationary.

As for the partial chronological Green's functions, these are also constructed from the product of the four Ψ-functions as follows. Only three of these are either chronological or anti-chronological. In other words, such Green's functions are the correlators of the type of $\langle\hat{\Psi}_4\hat{\tilde{T}}\{\hat{\Psi}_2^+\hat{\Psi}_3\hat{\Psi}_1^+\}\rangle$, $\langle\hat{T}\{\hat{\Psi}_4\hat{\Psi}_2^+\hat{\Psi}_3\}\hat{\Psi}_1^+\rangle$, $\langle\hat{T}\{\hat{\Psi}_4\hat{\Psi}_2^+\hat{\Psi}_3\}\hat{\Psi}_1^+\rangle$, etc.

To unify the notations, we redefine the two-particle Green's function by adding plus–minus signs to the number of the variable, as has been done in [17]

$$K(4,(s_4);2,(s_2);3,(s_3);1,(s_1)) \equiv K(4;2;3;1) \equiv \qquad (40)$$

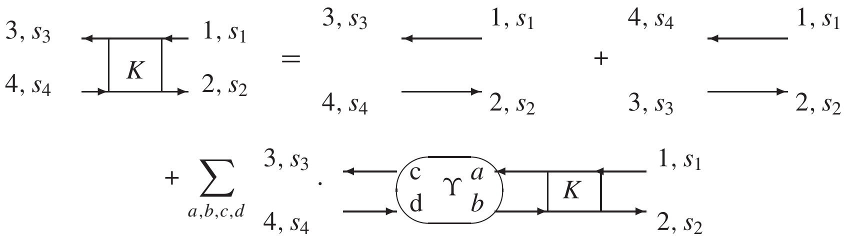


where symbols s_1, s_2, s_3 and s_4 can be either plus or minus.

When all signs s_i in equation (40) are minus or plus, then this is the standard K or anti-chronological $\tilde{K}$ two-particle Green's function. If only three signs are the same then it is the partial chronological Green's function $\hat{K}$. In other cases, the two-particle Green's functions are non-chronological K' functions [19].

It follows from equations (12)–(15) that the probability of photon production (13) is proportional to the two-particle Green's function $K(4(+), 2(-), 3(-), 1(+))$.

According to [19], the two-particle Green's functions satisfy the following equations:



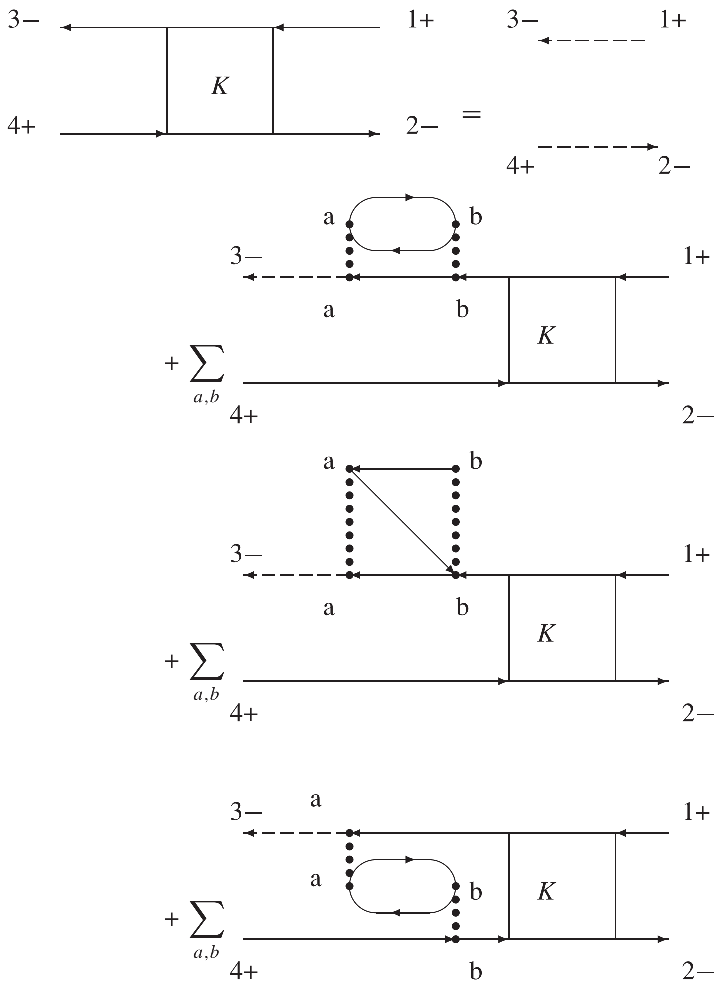
(41)

Here, Υ is the exact indecomposable diagram, which means the set of all the diagrams that cannot be cut by a vertical line so that this line only intersects two solid or dashed lines which correspond to the exact or free one-particle Green's functions in the Keldysh formalism [18, 19]. The parameters $a, b, c, d; a_1, b_1, c_1, d_1; a_2, b_2, c_2, d_2$ can take plus–minus signs. Summation in equation (41) is performed over all possible different combinations of these parameters for every sum symbol. Thus, we should carry out integration and summation over the variables corresponding to the points of the intersection of the ovals and the solid lines.

Let us consider that the particles are multiply scattered elastically in the matter. We assume that the concentrations of the particles and the interactions between them are such that the following inequality takes place

$$n|f|^2 \cdot \max\{|f|; \lambda\} \ll 1 \qquad (42)$$

where n is the number of particles per unit volume, f is the scattering amplitude of two particles in the matter, and λ is the wavelength of the particle. This inequality corresponds to what is known as the gas approximation. This means that we can be restricted to the first non-vanishing approximation with respect to the interaction between the particles in the matter. In this case, the two-particle Green's function $K(4(+); 2(-)|3(-); 1(+))$ satisfies the following equation



(43)

Here, the solid lines denote the standard one-particle Green's functions in the Keldysh diagram technique [18, 20], the dashed lines represent the Green's functions of free particles in the same formalism [18, 20], and the dotted lines imply interaction between the particles in the matter which is determined by the two-particle potential

$$\mathrm{i}U_{++}(X) \equiv \mathrm{i}U(\vec{r}_i - \vec{r}_j)\delta(t_1 - t_2) = {}^{+}\!\cdots\cdots\cdots{}^{+}$$
$$\mathrm{i}U_{--}(X) \equiv -\mathrm{i}U(\vec{r}_i - \vec{r}_j)\delta(t_1 - t_2) = {}^{-}\!\cdots\cdots\cdots{}^{-} \qquad (44)$$

Equation (43) can be rewritten analytically as follows

$$K(4(+); 2(-)|3(-); 1(+)) = K^{(0)}(4(+); 2(-)|3(-); 1(+)) + K^{(0)}(3(+); 2(-)|4(-); 1(+)) + \sum_{a,b}\int \mathrm{d}X_a\, \mathrm{d}X_b\, \mathrm{d}X'_a\, \mathrm{d}X'_b \cdot \{G^{(0)}(3(-); X_a(a))U_a(X_a - X'_a)U_b(X_b - X'_b)G(X_a(a); X_b(b))$$

$$\cdot G(X'_a(a); X'_b(b)) \cdot G(X'_b(b); X'_a(a)) K(4(+); 2(-)|X_b(b); 1(+))$$
$$+ G^{(0)}(3(-); X_a(a)) U_a(X_a - X'_a) U_b(X_b - X'_b) G(X_a(a); X_b(b))$$
$$\cdot G(X'_a(a); X_b(b)) \cdot G(X'_b(b); X'_a(a)) K(4(+); 2(-)|X_b(b); 1(+))$$
$$- G^{(0)}(3(-); X_a(a)) U_a(X_a - X'_a) U_b(X_b - X'_b) G(4(+); X_b(b))$$
$$\cdot G(X'_a(a); X'_b(b)) \cdot G(X'_b(b); X'_a(a)) K(X_b(b); 2(-)|X_a(a); 1(+))\} \qquad (45)$$

where $G(X_a(a); X_b(b))$ and $G^{(0)}(X_a(a); X_b(b))$ are the one-particle Green's functions for the interacting and free particles in the Keldysh formalism [18, 20], respectively, and a, b are plus or minus signs.

Moreover, we assume that the static approximation [21] for the description of the particle movement in the matter takes place. This approximation means that the matter returns to the initial state just after any individual collision of two particles in the medium. In other words, we assume that the particle producing photons undergoes pair collisions with other particles in the matter which are the static scatterers approximately. From the point of view of the diagram equation (43), this means tending all loops to the point in equation (43).

We assume that the emitting particle is a fermion with $s = 1/2$. Let us act by the Dirac operator $(\hat{p} - m)$, where m is the particle mass, on the variable whose number is '3' in equation (43). As a result, after the transfer to the momentum representation we go to the some equation which we write by convention as

$$\mathcal{F}_3(K(4(+); 2(-)|3(-); 1(+))) = 0 \qquad (46)$$

where the index '3' means the number of a variable.

Proceeding in the same way, we obtain similar equations with respect to the variables '1', '2' and '4':

$$\mathcal{F}_i(K(4(+); 2(-)|3(-); 1(+))) = 0 \qquad \mathrm{i} = 1, 2, 4. \qquad (47)$$

Then, we subtract the equation $\mathcal{F}_3(K(4(+); 2(-)|3(-); 1(+))) = 0$ from $\mathcal{F}_1(K(4(+); 2(-)|3(-); 1(+))) = 0$ and $\mathcal{F}_2(K(4(+); 2(-)|3(-); 1(+))) = 0$ from $\mathcal{F}_4(K(4(+); 2(-)| 3(-); 1(+))) = 0$. As a result we go to the two kinetic-like equations. In order to go to the approximation of the matter with static scatterers, we tend the loops to the point in equation (43).

After this, ignoring the 'mixing' of the spin variables due to the particle interaction in the matter[5], and going from the momentum representation of the two-particle Green's functions K to the function $F_{\vec{k}}(\vec{p}'; \vec{p}'; t_2; \vec{p}; \vec{p}; t_1)$ according to the formulae (15)–(17) we derive the equations (19) and (20).

References

[1] Landau L D and Pomeranchuk I Ya 1953 *Dokl. Akad. Nauk SSSR* **92** 535
[2] Landau L D and Pomeranchuk I Ya 1953 *Dokl. Akad. Nauk SSSR* **92** 735
[3] Migdal A B 1954 *Dokl. Akad. Nauk SSSR* **96** 49
[4] Migdal A B 1956 *Phys. Rev.* **103** 1811
[5] Ter-Mikhaekyan M L 1954 *Dokl. Akad. Nauk SSSR* **94** 1033
[6] Galitskii V M and Yakimetsl V V 1964 *Zh. Eksp. Teor. Fiz.* **46** 1066 *(Engl. transl. 1964 Sov. Phys.–JETP* **19** 723)
[7] Gol'dman I I 1960 *Zh. Eksp. Teor. Fiz.* **38** 1866 *(Engl. transl. 1960 Sov. Phys.–JETP* **11** 1341)
[8] Pafomov V E 1965 *Sov. JETP* **20** 353
[9] Baier R, Dokshitzer Yu L, Muller A H and Schiff D 1996 *Nucl. Phys.* B **478** 577

[5] Taking this effect into account leads to the small corrections of the order of $m/E \ll 1$ for the emission spectrum by ultra-relativistic particles [4].

[10] Zakharov B G 1996 *Russian JETP Lett.* **63** 952

[11] Zakharov B G 1996 *Russian JETP Lett.* **64** 781

[12] Klein S 1998 Suppression of bremsstrahlung and pair production due to environmental factors *Rev. Mod. Phys.* **77** 1501

[13] Ternovskii F F 1960 *Sov. JETP* **39** 171

[14] Shul'ga N F and Fomin S P 1978 *Sov. JETP Lett.* **27** 117

[15] Shul'ga N F and Fomin S P 1996 *Rusiia JETP Lett.* **63** 873

[16] Bethe H and Heitler M 1934 *Proc. R. Soc.* A **146** 83

[17] Knoll J and Voskresensky D N 1996 *Ann. Phys., NY* **249** 532

[18] Lifshitz E M and Pitaevskii L P 1979 *Physical Kinetics* (Moscow: Nauka)
Lifshitz E M and Pitaevskii L P 1981 *Physical Kinetics* (Oxford: Pergamon)

[19] Koshelkin A V 1999 *Phys. Lett.* B **471** 202

[20] Keldysh L V 1964 *Zh. Eksp. Teor. Fiz.* **47** 1515 *(Engl. transl. Sov.–JETP* **20** 235)

[21] Gyulassy M and Wang X-N 1994 *Nucl. Phys.* B **420** 583